Modelling and Estimating the Clustering of Extreme Events

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Motivation


Home | UK | World | Business | Politics | Tech | Science | Health | Education | Entertain

UK | England | N. Ireland | Scotland | Alba | Wales | Cymru

Storm Desmond: Homes flooded and thousands without power

6 December 2015 | UK

Helicopter footage reveals the scale of widespread flooding in Cumbria as flood waters reach their peak in the wake of Storm Desmond

Tens of thousands of homes are without power after Storm Desmond caused severe flooding and travel disruption across northern England and parts of Scotland.

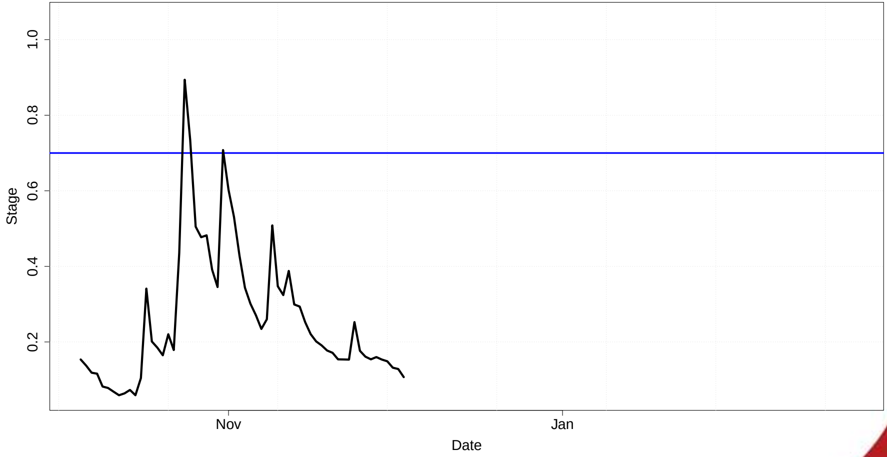
Credit: BBC NEWS

Credit: Barry Hankin, JBA Consulting

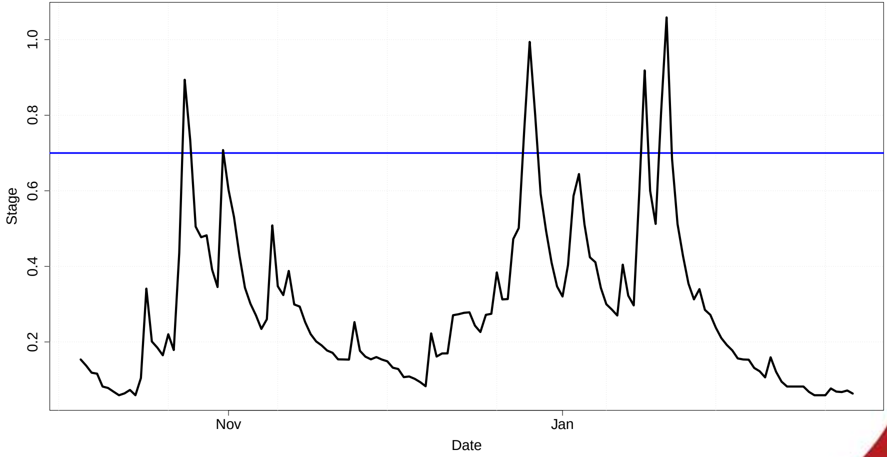
Motivation

- Regular occurrences of multiple extreme events being observed in the same season
- Large events are wrongly assumed to be independent and identically distributed
- Clustering of apparently independent exists due to local non-stationarity
- Develop a risk measure to characterise the heightened local risk of extreme events

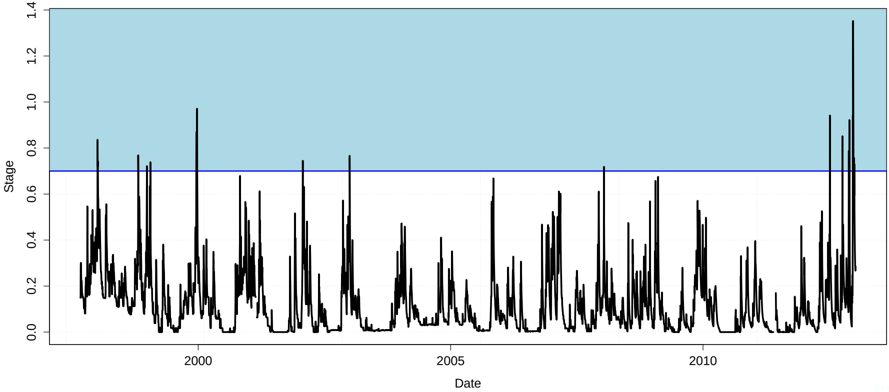
Stage Data



Stage Data



Modelling Strategy



Poisson Process

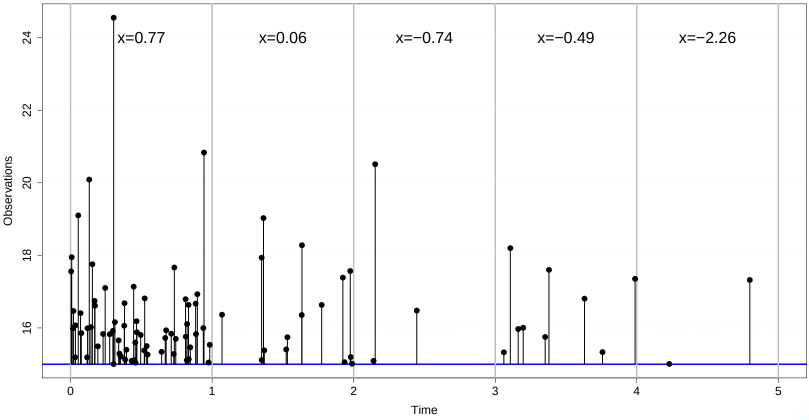
- Consider set of IID exceedances $Z_1, \ldots, Z_n$ above a sufficiently high threshold u
- Number of points N above the threshold u:
  $N \sim \text{Poisson}(\Lambda(z, u))$
- Characterise the exceedances in terms of the following parameters:
  - Location parameter $\mu \in \mathbb{R}$
  - Scale parameter $\sigma \geq 0$
  - Shape parameter $\xi \in \mathbb{R}$

$$G(z) = \exp\{-\Lambda(z, u)\} = \exp\left\{-\left[1 + \xi\left(\frac{z-\mu}{\sigma}\right)\right]_+^{-1/\xi}\right\}$$

Poisson Process

Covariate Effects

$$Z \sim \text{PP}(\mu(x) = 20 + 2x, \sigma = 2, \xi = 0.1), \text{ where } x \sim \text{N}(0, 1)$$

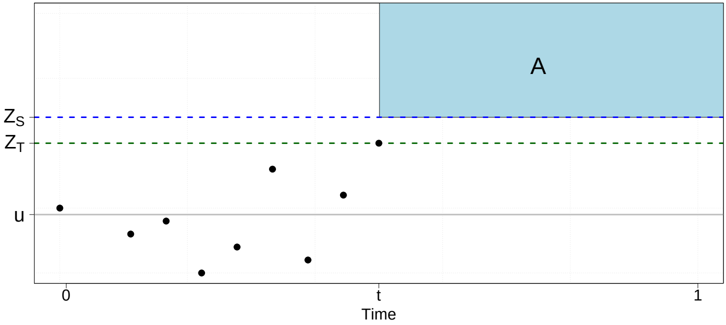


Definition of Relative Risk

Relative Risk	Definition
$R > 1$	An event of at least size z is R times more likely to occur if a 1 in T year event was already observed.
$R = 1$	There is no change in the risk of an event of at least size z occurring even if a 1 in T year event has already been observed.
$R < 1$	An event of at least size z is R times less likely to occur if a 1 in T year event was already observed.

Table: Definition of Relative Risk

Relative Risk Measure

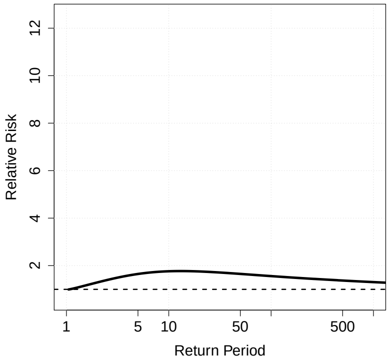


$$\text{Relative Risk} = \frac{P(M_{\lfloor tn \rfloor+1:n} > z_S | M_{1:\lfloor tn \rfloor} = z_T)}{P(M_{1:n} > z_S)}$$

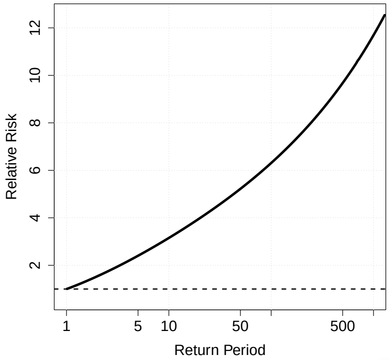
Simulated Example

1 in 100 year event and t=0.4

$$Z \sim \text{PP}(\mu(x) = 20 + 2x, \sigma = 2, \xi = \pm 0.1), \text{ where } x \sim \text{N}(0, 1)$$



Positive ξ



Negative ξ

Case Study

- Three hourly measurements of Stage from the River Harbourne from 1998 – 2012
- Focus of many studies by the Environment Agency
- Harbertonford has been flooded 21 times in the past 60 years
- Was flooded 6 times between 1998 and 2000

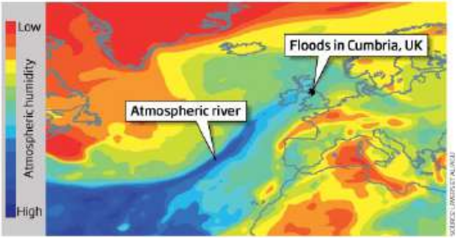
Credit: Environment Agency

Covariate Relationship

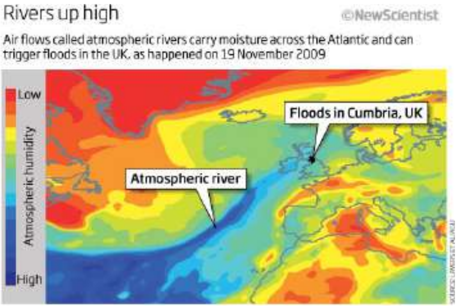
Rivers up high

©NewScientist

Air flows called atmospheric rivers carry moisture across the Atlantic and can trigger floods in the UK, as happened on 19 November 2009



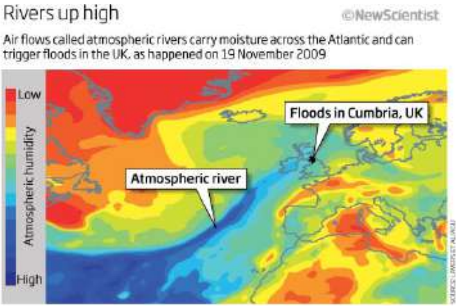
Covariate Relationship



Complexity:

- Difficult to detect large scale effects at a single location

Covariate Relationship



Complexity:

- Difficult to detect large scale effects at a single location
- Nearby rainfall gauges fail to explain changes in extremal behaviour

Covariate Relationship

Solution:

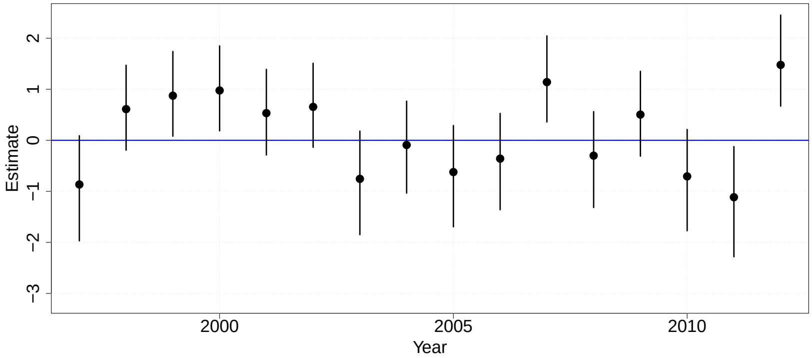
- Estimate the parameters using a Bayesian framework
- MCMC with a Metropolis Hastings algorithm
- Introduce a random effect in the location parameter

$$G_{z|s}(z) = \exp\left\{ -\left[1 + \xi \left(\frac{z - \mu(s)}{\sigma} \right) \right]_{+}^{-1/\xi} \right\},$$

where $\mu(s) = \mu_0 + \mu_1 s_i$ and $s_i \sim \mathrm{N}(0, 1)$

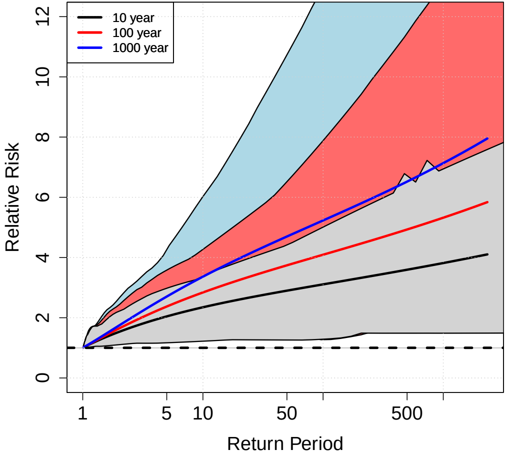
Random Effect Estimates

Estimates of s_i



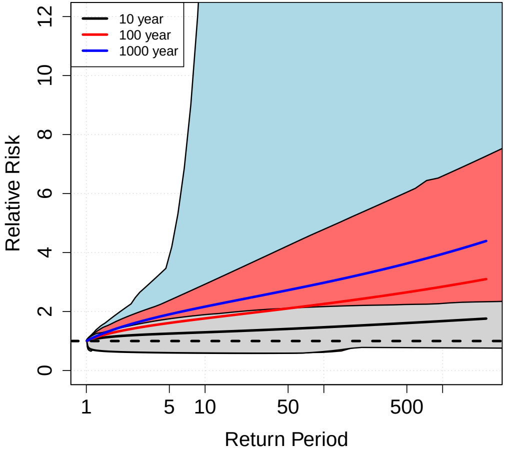
Relative Risk Estimate for Harbertonford

Extreme Event in October



Relative Risk Estimate for Harbertonford

Extreme Event in January



Conclusion and Further Work

- Development of risk measures which conveys the change in risk once extreme events have been observed

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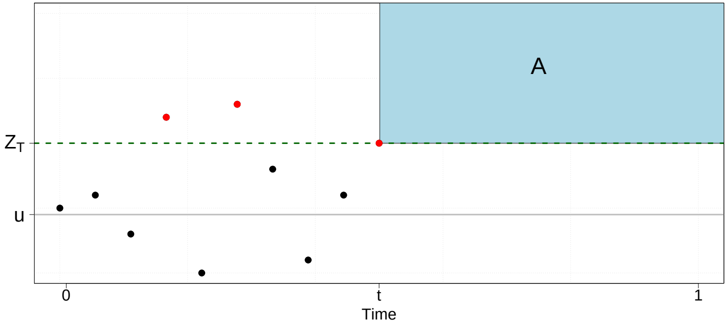
Any questions?

Second Risk Measure

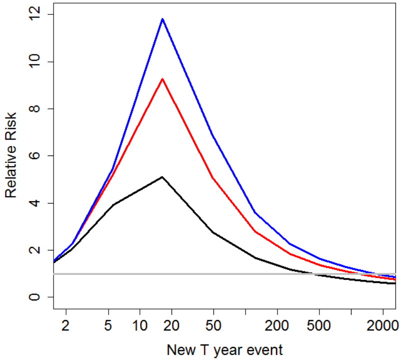


$$\text{Relative Risk 2} = \frac{P(M_{\lfloor tn \rfloor+1:n} > z_T | N_{1:\lfloor tn \rfloor} = n_1)}{P(M_{1:n} > z_T)}$$

Second Risk Measure

Simulated example

Positive ξ



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